

Duality of $N = 1$ Supersymmetric $SO(10)$ Gauge Theory with Matter in the Spinorial Representation

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We study $N = 1$ supersymmetric $SO(10)$ gauge theory with a field in the spinorial representation and N_f (≤ 8) fields in the defining representation. It is shown that this theory for $N_f = 7, 8$ has a dual description, which is $N = 1$ supersymmetric $SU(N_f - 5)$ gauge theory. Its matter content for $N_f = 7$ is different from the one for $N_f = 8$; for $N_f = 7$, it contains 8 fields in the anti-fundamental representation. For $N_f = 8$, a rank-2 symmetric tensor and one field in the fundamental representation appears in addition to them. This duality connects along the flat direction to the duality between chiral and vector gauge theory found by Pouliot.

The dynamical supersymmetry breaking (DSB) is a candidate to resolve the gauge hierarchy problem[1]. In the minimal SUSY standard model, the electroweak symmetry cannot be broken without the soft SUSY breaking terms. Thus the DSB may explain why the electroweak scale is so small compared with the Planck scale. However the Witten index[2] tells us that it is not easy to find models where the DSB can occur.

It has been discussed[3] that the DSB occurs in $SO(10)$ gauge theory with a matter in the spinorial representation **16**. Since this model is strongly interacting, the argument is indirect[3]. This is why this model is called non-calculable model. (for other example, see [4-7].) Recently Murayama[5] has introduced a field in the defining representation **10** into this model to have flat directions and has shown that the DSB occurs in this model. The presence of flat directions makes it easier to study the low energy behavior of $N = 1$ supersymmetric gauge theories, since we can apply the technique developed by Seiberg and his collaborators[8-11]. These technique has been revealing the non-perturbative properties of $N = 1$ supersymmetric gauge theories. (see [12], [13] and references therein.) Thus, after giving mass to the field in the defining representation and decoupling it, we can see that the DSB occurs in the original model, if we assume no phase transition occurs.

In this paper, we study the quantum moduli space of this model into which N_f fields in the defining representation are introduced. It is shown that the low energy theory of this model for $N_f = 7, 8$ has a dual description, which is $SU(N_f - 5)$ gauge theory. Its matter content for $N_f = 7$ is different from the one for $N_f = 8$; for $N_f = 7$, it contains 8 fields in the anti-fundamental representation. For $N_f = 8$, a rank-2 symmetric tensor and one field in the fundamental representation appears in addition to them. This duality connects along the flat direction to the duality between $SU(N_f - 5)$ chiral and $SO(7)$ vector gauge theory found by Pouliot[14].

Hopefully this result may shed some light toward the solution for the gauge hierarchy problem.

SO(10) gauge theory with a spinorial rep. and N_f defining reps.

We consider $N = 1$ supersymmetric $SO(10)$ gauge theory with a field ψ_α ($\alpha = 1, \dots, 16$) in the spinorial representation **16** and N_f fields H_A^i ($A = 1, \dots, 10; i = 1, \dots, N_f$) in the defining representation **10**. This theory is asymptotically free, if $N_f < 22$. The global symmetries are $SU(N_f) \times U(1)_F \times U(1)_R$. In our convention, the fields transform under these $U(1)$ symmetries in the following way:

$$\begin{aligned} U(1)_R : \quad & \psi_\alpha(\theta) \rightarrow \psi_\alpha(e^{-i\omega}\theta), \\ & H_A^i(\theta) \rightarrow e^{i\frac{N_f-6}{N_f}\omega} H_A^i(e^{-i\omega}\theta). \\ U(1)_F : \quad & \psi_\alpha(\theta) \rightarrow e^{-iN_f\omega} \psi_\alpha(\theta), \\ & H_A^i(\theta) \rightarrow e^{2i\omega} H_A^i(\theta). \end{aligned} \tag{1}$$

1. $N_f = 8$ and Magnetic $SU(3)$ gauge theory

We begin to consider the low energy effective theory for $N_f = 8$. All the gauge invariant operators can be generated by the operators

$$\begin{aligned} M^{ij} &= H_A^i H_A^j, \\ Y^i &= \psi^c \gamma_A \psi H_A^i, \\ Q_{e[i_1 i_2 i_3]} &= \epsilon_{i_1 i_2 i_3 j_1 \dots j_5} H_{A_1}^{j_1} \dots H_{A_5}^{j_5} \psi^c \gamma_{A_1 \dots A_5} \psi, \end{aligned} \tag{2}$$

where ψ^c is the charge conjugation of ψ . In absense of any superpotential at tree level, the flat directions are parametrized by these operators [15], [16] with classical constraints

$$\begin{aligned} Q_{e[i_1 i_2 i_3]} Q_{e[j_1 j_2 j_3]} &= -\frac{1}{6} Y^m M^{k_1 l_1} M^{k_2 l_2} \left[\epsilon_{m k_1 k_2 i_1 i_2 i_3 j_1 j_2} Q_{e[j_3 l_1 l_2]} \right. \\ &\quad \left. + (\text{permutation of } j_1, j_2 \text{ and } j_3) \right] \\ &\quad + \frac{1}{5!} \epsilon_{i_1 i_2 i_3 k_1 \dots k_5} \epsilon_{j_1 j_2 j_3 l_1 \dots l_5} M^{k_1 l_1} \dots M^{k_4 l_4} Y^{k_5} Y^{l_5}, \\ Y^i Q_{e[i j_1 j_2]} &= 0. \end{aligned} \tag{3}$$

These constraints are not modified by any quantum corrections. This low energy spectrum M^{ij} , Y^i and Q_e does not saturate the 't Hooft anomaly matching conditions[17]. In the quantum theory, the low energy effective superpotential cannot be written in terms of M^{ij} , Y^i and Q_e alone without any singularities. We suspect from these facts that there appear new light degrees of freedom, as SUSY QCD for $N_f \geq N_c + 2$ [11].

In fact, though it is a conjecture, this quantum theory has a dual description which we call ‘magnetic theory’; $SU(3)$ gauge theory with a symmetric tensor S_{ab} in **6**, a field q_a in the fundamental representation **3**, 8 fields \bar{q}_i^a in the anti-fundamental representation $\bar{\mathbf{3}}$ and singlets M^{ij} , Y^i . The superpotential in the magnetic theory is

$$W_m = \frac{1}{\mu_1^2} M^{ij} \bar{q}_i^a \bar{q}_j^b S_{ab} + \frac{1}{\mu_2^2} Y^i \bar{q}_i^a q_a + \det S, \quad (4)$$

where dimensionful parameters μ_1 and μ_2 are introduced to give the ‘meson’ fields M^{ij} , Y^i the same mass dimensions as those in the ‘electric’ theory. This theory is asymptotically free. Under the global symmetries $SU(N_f) \times U(1)_F \times U(1)_R$, the fields S_{ab} , q_a , \bar{q}_i^a , M^{ij} and Y^i transform as $(\mathbf{1}, 0, \frac{2}{3})$, $(\mathbf{1}, 16, \frac{4}{3})$, $(\bar{\mathbf{8}}, -2, \frac{5}{12})$, $(\mathbf{36}, 4, \frac{1}{2})$ and $(\mathbf{8}, -14, \frac{1}{4})$, respectively.

As a consistency check on this duality, the ’t Hooft anomaly matching conditions[17] are satisfied. The gauge invariant operators M^{ij} , Y^i and $Q_{e[i_1 i_2 i_3]}$ in the electric $SO(10)$ theory correspond to the operators M^{ij} , Y^i and $Q_{m[i_1 i_2 i_3]} = \epsilon_{a_1 a_2 a_3} \bar{q}_{i_1}^{a_1} \bar{q}_{i_2}^{a_2} \bar{q}_{i_3}^{a_3}$ in the magnetic $SU(3)$ theory. The other gauge invariant operators in the magnetic theory can be written by combining the above operators M^{ij} , Y^i and Q_m or identically vanish, through the equations of motion from the superpotential W_m .

At the point $\langle M^{88} \rangle \neq 0$ and $\langle Y^8 \rangle \neq 0$ on the moduli space, the electric $SO(10)$ gauge group is broken to $SO(7)$ by the vacuum expectation value of H_A^8 and ψ_α , and the other 7 fields H_A^i in the defining representation turn into those in the spinorial representation **8** under the unbroken $SO(7)$. (see the appendix in [5].) On the other hand, in the magnetic theory, the gauge group $SU(3)$ remains unbroken and the fields \bar{q}_8^a and q_a decouple at the energy scale below $\frac{1}{\mu_2^2} \langle Y^8 \rangle$. Solving the equations of motion, we obtain the low energy superpotential $W_{\text{eff}} = \sum_{i,j=1}^7 \frac{1}{\mu_1^2} M^{ij} \bar{q}_i^a \bar{q}_j^b S_{ab} + \det S$. Thus the present system just reduces to Pouliot’s model[14] in which the duality $SO(7) \leftrightarrow SU(3)$ was found. This is a non-trivial check on the duality of the model under consideration.

2. $N_f = 7$

For $N_f = 7$, the gauge invariant operators are the same as those for $N_f = 8$ except for Q_e ; $Q_{e[i_1 i_2]} = \epsilon_{i_1 i_2 j_1 \dots j_5} H_{A_1}^{j_1} \dots H_{A_5}^{j_5} \psi^c \gamma_{A_1 \dots A_5} \psi$, instead of $Q_{e[i_1 i_2 i_3]}$. The classical constraints among these operators are

$$\begin{aligned} Q_{e[i_1 i_2]} Q_{e[j_1 j_2]} &= -\frac{1}{6} Y^m \epsilon_{m i_1 i_2 j_1 j_2 k_1 k_2} Q_{e[l_1 l_2]} M^{k_1 l_1} M^{k_2 l_2} \\ &\quad + \frac{1}{5!} \epsilon_{i_1 i_2 k_1 \dots k_5} \epsilon_{j_1 j_2 l_1 \dots l_5} M^{k_1 l_1} \dots M^{k_4 l_4} Y^{k_5} Y^{l_5}, \\ Y^i Q_{e[ij]} &= 0. \end{aligned} \quad (5)$$

These constraints remain to hold even in the quantum theory. The light degrees of freedom M^{ij} , Y^i and Q_e again do not saturate the 't Hooft anomaly matching conditions[17].

In the electric theory for $N_f = 8$, after giving a mass to the field H_A^8 and decoupling it, we have the electric theory for $N_f = 7$. Alternatively, assuming the above duality for $N_f = 8$, adding a mass term mM^{88} to the magnetic superpotential (4) and using the equations of motion, we can see that the gauge symmetry $SU(3)$ breaks to $SU(2)$ by $\langle \bar{q}_8^3 \rangle \neq 0$ and the symmetric tensor S_{ab} become massive. After integrating out the field S_{ab} , we find the result proportional to $M^{ij} M^{kl} \bar{q}_i^a \bar{q}_k^b \epsilon_{ab} \bar{q}_j^c \bar{q}_l^d \epsilon_{cd}$ in the superpotential. In addition, by the holomorphy and the symmetries[8], the term $\det M (\frac{1}{M})_{kl} Y^k Y^l$ can be induced in the superpotential. Summing up these contributions, we obtain the following low energy superpotential:

$$W_{\text{eff}} = \frac{1}{\Lambda_7^{15}} \left[\det M \left(\frac{1}{M} \right)_{kl} Y^k Y^l - \frac{\mu^{10}}{2!} M^{ij} M^{kl} \bar{q}_i^a \bar{q}_k^b \epsilon_{ab} \bar{q}_j^c \bar{q}_l^d \epsilon_{cd} \right] + \frac{1}{\mu_2^2} Y^i \bar{q}_i^a q_a \quad (6)$$

with i, j, k and l now running from 1 to 7 and a, b, c and d running from 1 to 2. The scale Λ_7 is the dynamical scale in the electric theory, which is connected to the dynamical scale $\tilde{\Lambda}_{2,7}$ in the magnetic theory as $\Lambda_7^{15} [\tilde{\Lambda}_{2,7}^2]^2 = \langle S_{33} \rangle^3 \mu_1^{12} \mu_2^4$. The dimensionful parameter μ is defined by $\mu^5 = -\langle S_{33} \rangle \mu_1^4 \mu_2^2 / \tilde{\Lambda}_{2,7}^2$. The composite operator $Q_{m[ij]} = \mu^5 \bar{q}_i^a \bar{q}_j^b \epsilon_{ab}$ can be identified with $Q_{e[ij]}$ in the electric theory. The operator $\bar{q}_i^a q_a$ is vanishing as we can see from the equation of motion for Y^i and the other gauge invariant operators can be constructed by combining the operators M^{ij} , Y^i and Q_m . Thus the correspondence of the gauge invariant operators between the electric and the magnetic theory remains for $N_f = 7$. From the equations of motion for M^{ij} and q_a , the constraints(5) in the electric theory are partly reproduced from the magnetic superpotential(6).

This theory is asymptotically free. (Note that it is not asymptotically free until integrating out the symmetric tensor S_{ab} .) We can verify that these fields saturate the 't Hooft anomaly matching conditions[17].

At the point $\langle M^{77} \rangle \neq 0$, $\langle Y^7 \rangle \neq 0$ on the moduli space, the electric gauge group $SO(10)$ breaks to $SO(7)$ and the matter content becomes 6 fields in the spinorial representation. For the magnetic theory, on the other hand, \bar{q}_7^a and q_a become massive and should be integrated out. By the holomorphy and the symmetries[8], there is another allowed contribution proportional to $\epsilon^{7k_1 \dots k_6} \bar{q}_{k_1}^{a_1} \bar{q}_{k_2}^{a_2} \epsilon_{a_1 a_2} \dots \bar{q}_{k_5}^{a_5} \bar{q}_{k_6}^{a_6} \epsilon_{a_5 a_6}$. The resultant superpotential

is proportional to $\det M - \frac{1}{2} M^{ij} M^{kl} B_{ik} B_{jl} - \text{Pf } B$ with $B_{ij} \equiv Q_{m[ij]}/\langle Y^7 \rangle$. This is exactly the same superpotential as was found by Pouliot[14], who has shown that these composite fields M^{ij} and B_{ij} saturate the 't Hooft anomaly matching conditions[17] for the microscopic $SO(7)$ gauge theory. Conversely the equations of motion from this superpotential reproduce the classical constraints (5) in the electric theory. This is interesting because the classical relations in the electric theory is the result by the non-perturbative dynamics in the magnetic theory. Therefore the electric $SO(10)$ gauge theory and the magnetic $SU(2)$ reduce to the same low energy theory in the infrared at this point $\langle M^{77} \rangle \neq 0$, $\langle Y^7 \rangle \neq 0$ on the moduli space.

Thus our conjecture follows that the $SU(2)$ gauge theory with the fields \bar{q}_i^a , q_a , M^{ij} and Y^i and the above superpotential (6) describes the low energy dynamics of the $SO(10)$ gauge theory for $N_f = 7$.

3. $1 \leq N_f \leq 6$

For $N_f = 6$, the flat directions can be described by the gauge invariant operators M^{ij} , Y^i and $Q_i = \epsilon_{ij_1 \dots j_5} \psi^c \gamma_{A_1 \dots A_5} \psi H_{A_1}^{j_1} \dots H_{A_5}^{j_5}$, as we have seen for $N_f = 7$ and 8. The classical constraints are

$$\begin{aligned} Q_i Q_j &= \frac{1}{5!} \epsilon_{ik_1 \dots k_5} \epsilon_{jl_1 \dots l_5} M^{k_1 l_1} \dots M^{k_4 l_4} Y^{k_5} Y^{l_5}, \\ Y^i Q_i &= 0. \end{aligned} \tag{7}$$

By $U(1)_R$ symmetry, no effective superpotential is dynamically generated. The quantum theory has the moduli space of the degenerate ground states. The massless spectrum M^{ij} , Y^i and Q_i does not saturate the 't Hooft anomaly matching conditions[17]. Symmetry argument suggests that the classical constraints (7) have the possibility to be modified quantum mechanically into

$$\begin{aligned} \det M \left(\frac{1}{M} \right)_{kl} Y^k Y^l - M^{kl} Q_k Q_l &= \Lambda_6^{16}, \\ Y^i Q_i &= 0, \end{aligned} \tag{8}$$

with Λ_6 being the dynamical scale in the electric theory for $N_f = 6$.

We show consistency checks on this quantum constraints (8). First, along the flat directions $\langle M^{66} \rangle \neq 0$, $\langle Y^6 \rangle \neq 0$ by which the electric $SO(10)$ gauge theory turns into the $SO(7)$ gauge theory with 5 spinorial representations, the quantum constraints (8) reduce to $\det M - M^{ij} B_i B_j = \Lambda_{SO(7)}^{10}$ with i, j running from 1 to 5, which is the quantum

constraint found for $N_f = 5$ in [14], if we identify $B_i = Q_i/\langle Y^6 \rangle$ and $\Lambda_{SO(7)}^{10} = \Lambda_6^{16}/\langle Y^6 \rangle^2$. Second, adding a mass term mM^{77} to the superpotential (6) in the dual magnetic theory and integrating out massive fields, we find the magnetic gauge group $SU(2)$ is completely broken by $\langle \bar{q}_7^2 \rangle \neq 0$ and obtain the quantum constraints (8) under the one-loop matching condition $\Lambda_6^{16} = m\Lambda_7^{15}$. Here the operator Q_i turns out to be $\mu^5 \langle \bar{q}_7^2 \rangle \bar{q}_i^1$.

The quantum constraints (8) are implemented by the following effective superpotential for $N_f = 6$:

$$W_{\text{eff}} = X \left(\det M \left(\frac{1}{M} \right)_{kl} Y^k Y^l - M^{kl} Q_k Q_l - \Lambda_6^{16} \right) + LY^i Q_i \quad (9)$$

where X and L are the Lagrange multiplier fields.

For $N_f = 5$, by adding a mass term mM^{66} to the above superpotential (9) and integrating out massive fields, we can find the low energy effective superpotential for $N_f = 5$

$$W_{\text{eff}} = \left[\frac{\Lambda_5^{17}}{(M^4 \cdot Y^2) - Q^2} \right] \quad (10)$$

with the scale $\Lambda_5^{17} = m\Lambda_6^{16}$ and the operator $Q = Q_6$, where $(M^4 \cdot Y^2) = \det M(\frac{1}{M})_{kl} Y^k Y^l$. The singularity at the point $(M^4 \cdot Y^2) - Q^2 = 0$ indicates that the gauge symmetry is not completely broken and the subgroup remains at this point.

By the $SO(7)$ flat directions $\langle M^{55} \rangle \neq 0$, $\langle Y^5 \rangle \neq 0$, the superpotential (10) reproduces the effective superpotential $\Lambda_{SO(7)}^{11}/[\det M - B^2]$ for $N_f = 4$ in [14] under the identification $B = Q/\langle Y^5 \rangle$ and $\Lambda_{SO(7)}^{11} = \Lambda_5^{17}/\langle Y^5 \rangle^2$.

For $1 \leq N_f \leq 4$, by the holomorphy and the symmetries[8], the following low energy effective superpotential W_{eff} is allowed to arise:

$$W_{\text{eff}} = (6 - N_f) \left[\frac{\Lambda_{N_f}^{22-N_f}}{(M^{N_f-1} \cdot Y^2)} \right]^{\frac{1}{6-N_f}}, \quad (11)$$

where $(M^{N_f-1} \cdot Y^2) = \det M(\frac{1}{M})_{kl} Y^k Y^l$. Indeed, adding mass terms and decoupling massive fields, we find that these coefficients are consistently determined under the one-loop matching condition $\Lambda_{N_f}^{22-N_f} = m\Lambda_{N_f+1}^{21-N_f}$. Furthermore by adding a mass term mM^{55} to the low energy superpotential (10) for $N_f = 5$ and integrating out massive fields, we obtain the superpotential (11) for $N_f = 4$.

Thus the theory for $1 \leq N_f \leq 5$ has no vacuum, until we properly add superpotentials at tree level to it.

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